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Coordinates as determined by Side Scan Sonar

- Theory and Application-

J.M. Preston

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Research and Development Branch
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COORDINATES AS DETERMINED BY SIDE-SCAN SONAR
THEORY AND APPLICATION

ERRATUM

1. In Equation 14, replace 1^2 by λ^2 (three occurences)

2. In the Glossary, replace V_x, V_y, V_z by x_b, y_b, z_b

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Technical Memorandum 88-2

Coordinates as Determined by Side-Scan Sonar - Theory and Applications

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ABSTRACT

A central issue in minehunting is the accuracy with which the coordinates of a mine-like object can be determined. This survey process is done in stages, first locating the ship and then locating the object with respect to the sonar. If side-scan sonar is used, there is the additional stage of determining the vector between the ship and the towfish. The equations which determine the chart coordinates of a bottom object from the 14 variables which describe the side-scan sonar deployment which imaged that object are derived and presented. The improvements in the accuracy of the coordinates which could be achieved by improving the accuracy with which any variable is measured are then predicted. Coordinate data acquired during MINEX 87 is used to calculate the unmeasured angular variables which characterized that deployment.

TABLE OF CONTENTS

ABSTRACT	iii
INTRODUCTION	1
CALCULATION OF THE UTM COORDINATES OF BOTTOM OBJECTS FROM OPERATIONAL VARIABLES	2
Introduction	2
Object in Towfish Frame	4
Towfish in Ship's Coordinate System	6
Vector between the Known Points on the Ship	9
Object Position in Geographic Coordinates	10
ANALYSIS OF ERRORS IN COORDINATES DETERMINED BY SIDE-SCAN SONAR	11
Introduction	11
Towfish Frame of Reference	13
Ship Frame of Reference	13
World Frame of Reference (UTM)	13
Total Variance in Position	15
Application to a Typical Side-Scan Deployment	15
DERIVATION OF THE ANGULAR VARIABLES OF TAP DURING MINEX 87	20
Introduction	20
Coordinates of Bottom Objects	22
Analysis	26
Comparision of Derived Angles with Observation	35
CONCLUSIONS	37
GLOSSARY OF VARIABLES	38

INTRODUCTION

A central issue in minehunting is the accuracy with which the position of a mine-like object (MLO) on the sea bottom can be determined. One use for the coordinates of the objects is comparison with route survey (i.e. pre-mining) data or imagery. Inaccuracies can increase the number of mismatches and thereby reduce the value of this approach. Accurate coordinates are also essential for returning to the object for classification and for disposal, should it be a live mine. While some minefield clearance strategies avoid the latter requirement by prosecuting all contacts as they are detected these strategies have other drawbacks. In particular, all the minefield-clearance procedures being considered for the Canadian Forces rely on knowing accurate locations of MLOs.

The challenge in accurate determinations of bottom object locations is that a combination of navigational and positioning systems is required. If the sonar is hull-mounted, the ship position and attitude, the range and bearing to the target, and the water depth must all be known. With a side-scan sonar, as is being considered for the CF, the attitude and position of the towfish are also relevant. Thus a considerable number of variables are required to completely describe the situation at the time the MLO is imaged. The use of these variables to calculate the coordinates of the bottom object requires a number of coordinate transformations. The resulting expressions do not seem to be available elsewhere and are therefore presented in the first part of this Memorandum. Because most of the variables describe the towfish attitude or position (TAP), it is convenient to refer to these expressions as TAP theory.

Two applications of TAP theory follow. The first addresses the effects of errors in each of the measured variables on the final position accuracy of the bottom object. A series of graphs illustrate the benefits or dangers of increasing or degrading the accuracy of each of the measurements involved. The second application is to the images gathered during a recent minehunting exercise conducted in the approaches to Halifax,

MINEX 87. During this exercise not all the variables necessary to completely describe the deployment were measured. However because the entire area was surveyed several times, the sonar records contain numerous images of each prominent bottom feature, each of which yielded a pair of coordinates. TAP theory was applied to these lists of coordinates to calculate the mean values of the more important unmeasured variables, namely towfish pitch, roll, and yaw, and the angle from the ship to the towfish.

Any coordinate system could have been used. In MCM, DREP practice is to use the Universal Transverse Mercator (UTM) projection, or a local grid conforming to UTM, and that practice has been continued here.

A glossary of all the relevant variables is included at the end of this Memorandum as Table 3.

CALCULATION OF THE UTM COORDINATES OF BOTTOM OBJECTS FROM OPERATIONAL VARIABLES

Introduction

Explicit expressions are presented in this Section for the UTM coordinates of a bottom object imaged by side-scan sonar. The operational variables considered include all those relevant to a towfish equipped with side-scan sonar, namely pitch, roll, and yaw of the towfish and of the ship, position of the towfish with respect to the ship as measured by an acoustic baseline system (e.g. ultra-short baseline, USBL), towfish depth and height above the bottom, and the vector between the two known points on the ship (i.e. the locations of the navigational antenna and the acoustic baseline). The only assumptions which have been made are that the bottom is level and that sound travels in straight lines.

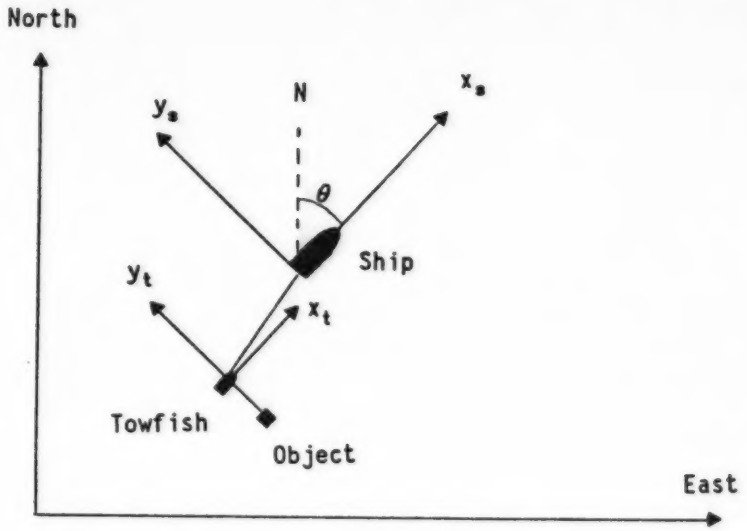

Figure 1. Coordinate systems used in this derivation.

The derivation proceeds progressively in coordinate systems based on the towfish, on the ship, and on geography (e.g. UTM coordinates). All three coordinate systems are shown in Figure 1. The subscript indicates the origin of the coordinate system being considered. The result we seek is (E_o, N_o), the Easting and Northing of the insonified object in UTM coordinates. It is important to note that the first two coordinate systems are centered on those vehicles but not, in general, oriented with them. In all coordinate systems the x-y plane is horizontal with the x axis along the ship's course (as distinct from the current heading), and the z axis vertically down. The ship's course, i.e. the bearing of the line being surveyed, is preferred as the direction of the x axis because it does not vary with time during the run of a scan line and thus provides a reference for both towfish and ship yaw. Primed versions of these frames, oriented along the vehicles, are used briefly.

The coordinate systems used here are left-handed. This selection resulted from retaining the normal identification of x with East and y with North while allowing z to be positive downward.

Object in Towfish Frame

If the towfish has zero pitch, roll, and yaw, the coordinates of an object at range r (one-way travel time / sound speed) are, in the towfish-centered coordinate system

$$x_t = 0, \qquad y_t = k\,[r^2 - h^2]^{1/2} \tag{1}$$

where h is the height of the towfish above the bottom and the index k is +1 for an object in the port sonar channel, -1 for the starboard side. To be completely precise, one should specify that the origin of the coordinate system is at the centre of the transducer at the instant that the sound reflects from the object.

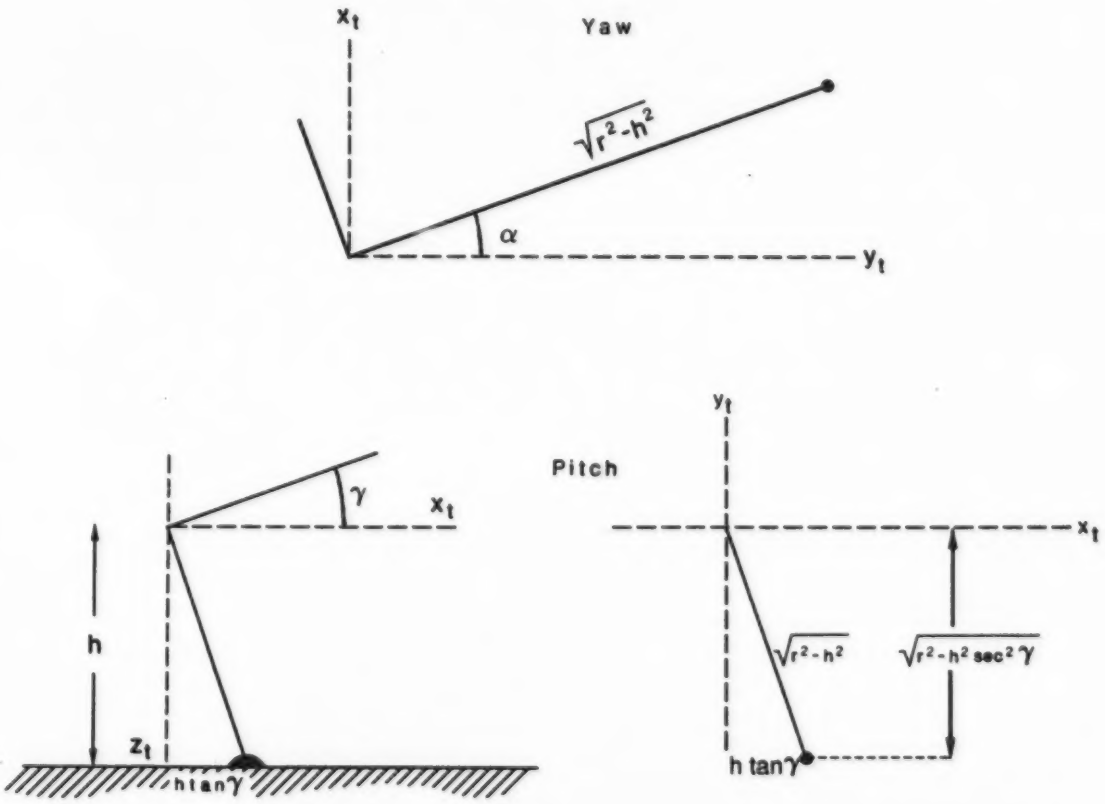

Figure 2. Illustrating the location of a bottom object at range r from a towfish at height h which is yawed through angle α or pitched through γ.

The towfish coordinate system (x_t, y_t, z_t) is centered on the towfish with the x axis horizontal and parallel to the ship's course, the y axis horizontal and the z axis vertical downwards. The towfish will, in general, have rotated by a yaw angle α_t (about the z axis), a roll angle β_t (about the x axis) and a pitch angle γ_t (about the y axis). The geometry of side-scan sonar makes the imaging process insensitive to moderate roll angles, because the sonar beam is broad in the y-z plane. Since rotation about the x axis has no effect on imaging, we may as well assume zero roll, and will do so. The effects of yaw and pitch are illustrated in Figure 2.

In this section we will calculate the coordinates of the object in the towfish coordinate system for a yawed and pitched towfish. The sonar equations (Eq. 1) apply in a coordinate system which has yawed and pitched with the towfish (x_t', y_t', z_t'). The transformation is

$$\begin{bmatrix} x_t' \\ y_t' \\ z_t' \end{bmatrix} = \begin{bmatrix} \cos\alpha_t \cos\gamma_t & \sin\alpha_t \cos\gamma_t & -\sin\gamma_t \\ -\sin\alpha_t & \cos\alpha_t & 0 \\ \cos\alpha_t \sin\gamma_t & \cos\alpha_t \sin\gamma_t & \cos\gamma_t \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} \tag{2}$$

The height, horizontal distance, and range, in the yawed and pitched frame, are a right-angled triangle, thus:

$$y_t'^2 + z_t'^2 = r^2 \tag{3}$$

and the object is directly abeam of the towfish:

$$x_t' = 0 \tag{4}$$

while the height above the bottom is known:

$$z_t = h \tag{5}$$

Eliminating one coordinate from these last three equations and solving the resulting quadratic gives:

$$x_t = -k \sin\alpha_t [r^2 - h^2 \sec^2\gamma_t]^{1/2} + h \cos\alpha_t \tan\gamma_t$$

$$y_t = k \cos\alpha_t [r^2 - h^2 \sec^2\gamma_t]^{1/2} + h \sin\alpha_t \tan\gamma_t \quad . \qquad (6)$$

Towfish in Ship's Coordinate System

Now we turn to the coordinate system centered on the ship, still with the x axis horizontal and along the ship's course and the y and z axes horizontal and vertically downward respectively. It is assumed that an acoustic positioning system is used to locate the towfish with respect to the ship. Because the acoustic baseline system measures distance and angles from its transducer, the origin of the ship coordinate system is best placed on the centre of the acoustic baseline.

An acoustic positioning system, as mounted on the ship, consists of three "point" transducers arranged in a right angle, in the nominally horizontal plane. They may be spaced by several wavelengths, in which case the differences in arrival times of a ping from the remote object are measured, or they may be spaced by a fraction of a wavelength, in which case phase differences are measured. In both cases the transit time of the ping, or of a two-way ping if a transponder is in use, gives the range to the remote vessel or object. Two angles can be calculated from the arrival-time or phase differences, one in the plane of the transducers and an angle of depression from that plane. These angles follow directly from the direction cosines of the sound ray on the axes defined by the transducers. Thus with the plane of the transducers lying horizontal and with the line through one pair of transducers facing fore and aft, the coordinates of the vector from the acoustic system to the remote object can be determined.

A full analysis of the accuracy of acoustic positioning systems is fairly complex, as indicated by the partial discussion in the next three paragraphs. For the present purposes, the accuracy given by the manufacturers will be used. This is usually quoted as a percentage of the slant range, the percentage varying with the size of the angles being measured.

Misalignment of the transducers with respect to the fore-aft line of the ship is usually determined by deploying a remote transducer at a known angle (e.g. directly off the stern). The resulting offsets are then applied to all future measurements.

Since the acoustic baseline is rigidly fixed to the ship, it actually measures the position of the remote vessel in a coordinate system which yaws, rolls, and pitches with the ship. Correction for yaw is done with the accuracy of the ship's gyro, and is considered explicitly in this document. Corrections for roll and pitch require inclinometers. Commercial systems use pendulous inclinometers, because of the cost of gyroscopically stabilized attitude sensors. The error in the correction is therefore from two sources, instrumental errors and the use of a non-inertial frame.

A further input to most commercial acoustic positioning systems is the depth of the towfish, as measured by a pressure transducer. The purpose of measuring depth is to improve the positioning accuracy, as illustrated in Figure 3a. This Figure is in the vertical plane containing the ship and the remote object. Without depth information, the object could be anywhere in the volume $2\Delta R \times 2R\Delta\phi \times 2R\Delta\delta$ (δ is the angle off astern, perpendicular to the plane of the Figure). Knowledge of the depth, d, with an error bound Δd, reduces the volume to the product of the shaded area shown and $2R\Delta\delta$. A potentially more valuable benefit of depth information would occur when temperature gradients cause refraction of the sound ray (the solid line in Figure 3b) as it rises to the ship. The acoustic system, operating without correction for refraction, would place the object in the volume $2\Delta R \times 2R\Delta\phi \times 2R\Delta\delta$, as before (Figure 3b). The disparity with the measured depth allows a correction for refraction to be made. However this topic cannot be analyzed here without detailed knowledge of the algorithims used, and these are not available for commercial reasons.

The coordinate system in which the acoustic baseline positioning system operates has the x-y plane horizontal and the z axis vertical downward, like our ship-centered system, but has its x axis along the ship's

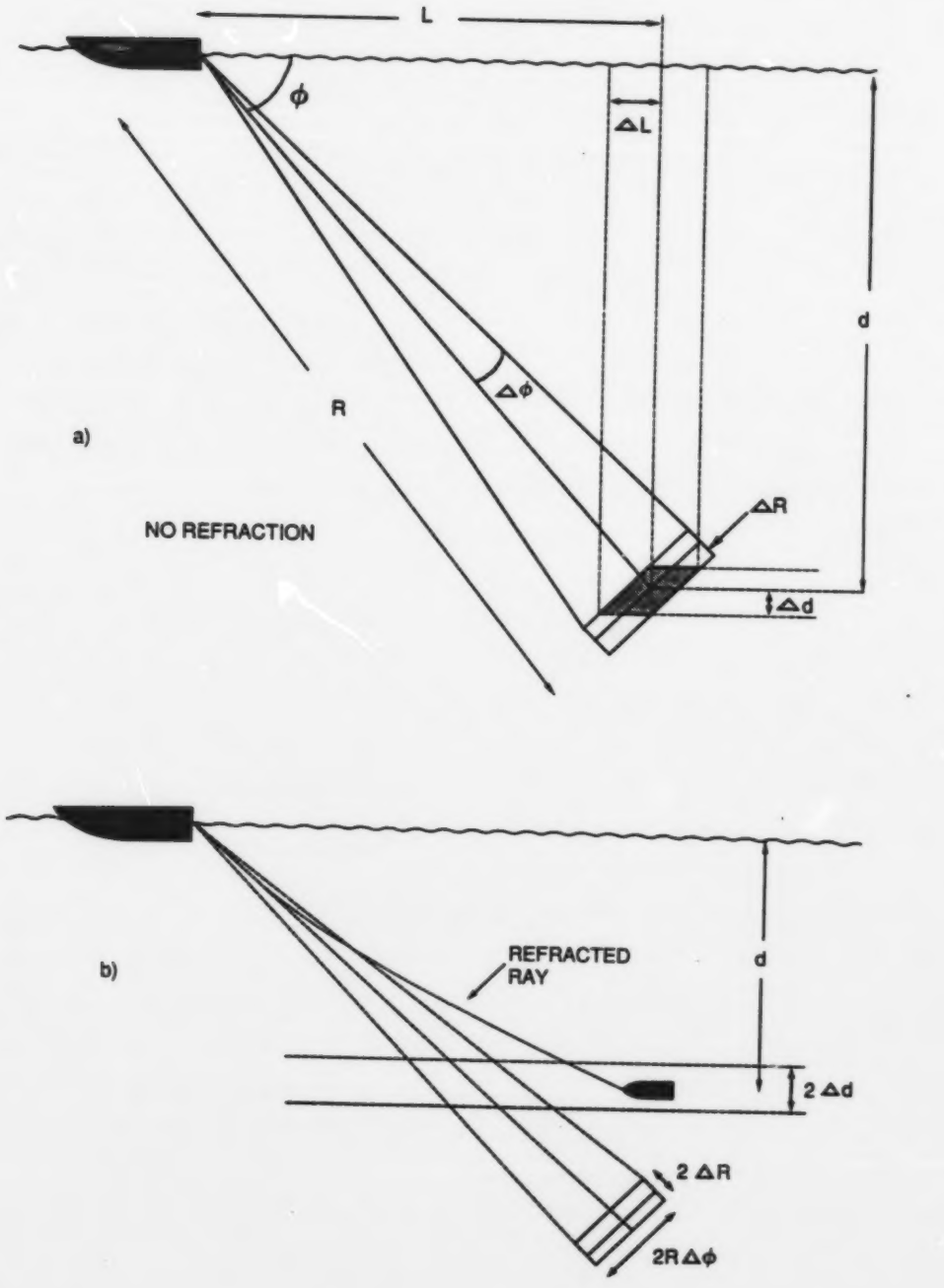

Figure 3. The use of a depth measurement to improve the accuracy of position determination by an acoustic positioning system. In part (a) there is no refraction and the improvement in the measurement of the layback is shown. In part (b) refraction causes an apparent disparity.

heading rather than the course. The two differ by the current value of the yaw. Therefore:

$$\begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} \cos\alpha_s & -\sin\alpha_s & 0 \\ \sin\alpha_s & \cos\alpha_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} + \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} \qquad (7)$$

where (B_x, B_y, B_z) is the towfish vector as measured by the acoustic positioning system and (x_s, y_s, z_s) is the position of the insonified object in the coordinate system centered on the acoustic baseline with the x axis on the ship's course. If the towfish is dead astern, then $(B_x, B_y, B_z) = (-L, 0, d)$, where L is the horizontal distance between the ship and the towfish, also called the layback, and d is the towfish depth.

Vector between the Known Points on the Ship

The next step is to derive the coordinates of the object in the coordinate system centered on the point in the ship at which the navigational antenna is located. It is the navigational antenna whose position is determined by the navigational system. We define a new coordinate system oriented as before (x axis along the ship's course, y axis horizontal, and z axis downward, so that transformations can be done by simple addition of components), with the origin at the navigational antenna.

In the coordinate system centered on the navigational antenna, the vector to the centre of the acoustic baseline in the absence of roll, pitch, and yaw is $(x_b, y_b, z_b) = (-D_x, D_y, D_z)$. The minus sign is included so that the distances, D, will generally be positive, that is, the baseline is usually aft of and below the navigational antenna. D_y is positive if the baseline is on the port side, zero if amidships. Attitude variations rotate the vector to the baseline centre through the same angles α_s, β_s, and γ_s considered above. The vector components transform as described by the normal coordinate transform, but with the signs of all angles reversed since it is the vector, not the coordinate system, whose rotation is being described. Thus:

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} \cos\alpha_s \cos\gamma_s & -\sin\alpha_s \cos\gamma_s & \sin\gamma_s \\ \cos\alpha_s \sin\beta_s \sin\gamma_s + \sin\alpha_s \cos\beta_s & -\sin\alpha_s \sin\beta_s \sin\gamma_s + \cos\alpha_s \cos\beta_s & -\sin\beta_s \cos\gamma_s \\ -\cos\alpha_s \cos\beta_s \sin\gamma_s + \sin\alpha_s \sin\beta_s & \sin\alpha_s \cos\beta_s \sin\gamma_s + \cos\alpha_s \sin\beta_s & \cos\beta_s \cos\gamma_s \end{bmatrix} \begin{bmatrix} -D_x \\ D_y \\ D_z \end{bmatrix} \tag{8}$$

and therefore

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} -D_x \cos\alpha_s \cos\gamma_s - D_y \sin\alpha_s \cos\gamma_s + D_z \sin\gamma_s \\ -D_x (\cos\alpha_s \sin\beta_s \sin\gamma_s + \sin\alpha_s \cos\beta_s) - D_y (\sin\alpha_s \sin\beta_s \sin\gamma_s - \cos\alpha_s \cos\beta_s) - D_z \sin\beta_s \cos\gamma_s \\ D_x (\cos\alpha_s \cos\beta_s \sin\gamma_s - \sin\alpha_s \sin\beta_s) + D_y (\sin\alpha_s \cos\beta_s \sin\gamma_s + \cos\alpha_s \sin\beta_s) + D_z \cos\beta_s \cos\gamma_s \end{bmatrix} \tag{9}$$

Object Position in Geographic Coordinates

Finally, we now express the image coordinates in a geographical reference frame. For imaging, only the Easting and Northing are important. If E_s and N_s are the coordinates of the ship's navigational antenna in the geographic system, and θ is the ship's course, then our final result is:

$$E_o = E_s + (x_b + x_s) \sin\theta - (y_b + y_s) \cos\theta$$

$$N_o = N_s + (x_b + x_s) \cos\theta + (y_b + y_s) \sin\theta \tag{10}$$

in which x_b and y_b are given in Eq. 9 and x_s and y_s in Eq. 7, while x_t and y_t, which enter into Eq. 7, are given in Eq. 6. All variables are listed in the Glossary, Table 3, located at the end of this Memorandum.

ANALYSIS OF ERRORS IN COORDINATES DETERMINED BY SIDE-SCAN SONAR

Introduction

The accuracy with which the coordinates of bottom objects can be determined is the key to successful geocoding of neighbouring images, and is also obviously important if it is necessary to further investigate an object. As we have seen, many operational variables are required in order to calculate the coordinates, and these variables, if known at all, are known to different accuracies. In this section the contribution of each of the errors to the total error in the coordinates will be calculated from TAP theory. The resulting expressions are applied to a typical case. Also, the improvement in the accuracy of the coordinates which would be generated by improvements in the accuracy of each of the operational variables will be derived.

Because of the number of variables, it was found convenient to consider first those which relate to the towfish frame of reference, then those which relate to the ship frame, and finally those which relate the ship frame to the UTM frame.

The propagation of errors through to a value V calculated as a function of measured values x_i is described by the Taylor expansion:

$$\Delta V \simeq \sum_i \left[\frac{\partial V}{\partial x_i} \right] \Delta x_i \qquad (11)$$

where the partial derivatives are taken with the other variables held constant at their mean values. The implicit approximation is that the second-order terms can be neglected. Provided the errors in the variables x_i are uncorrelated, the variance of V, σ_V^2 is

$$\sigma_V^2 \simeq \sum_i \left[\frac{\partial V}{\partial x_i}\right]^2 \sigma_{x_i}^2 \quad . \tag{12}$$

Thus the variance of any derived quantity, such as x_t, B_y, or E_o, can be found by calculating the partial derivatives of the quantities with respect to each of the variables with all variables at their mean values, and then performing the indicated summation. In particular, the variance in position of an object is the sum of the variances in the x and y coordinates. Also, the overall variance in each coordinate is the sum of the variances of that coordinate in each of the three frames, because the UTM coordinate is the sum of that coordinate in each of the three frames.

Towfish frame of reference

The variables in the towfish frame as defined above (x axis on ship's course, y and z axes horizontal and vertical) are those which enter into the calculation of x_t and y_t, namely the towfish yaw and pitch, the towfish height, and the range to the object. An additional error arises from the horizontal width of the sonar beam. The position variance in the towfish frame of reference is thus

$$\sigma_{tf}^2 = \sigma_{x_t}^2 + \sigma_{y_t}^2 + r^2\varepsilon^2 \tag{13}$$

where ε is the horizontal beamwidth. The terms can be derived by applying Eq. 12 to Eq. 6, resulting in nine terms since x_t and y_t each depend on four variables, r, h, α_t, and γ_t, plus the additional term due to the horizontal beamwidth. Two terms can be combined through a trigonometric identity, giving:

$$\sigma_{tf}^2 = \frac{r^2}{l^2}\sigma_r^2 + \left[\frac{h^2}{l^2}\sec^4\gamma_t + \tan^2\gamma_t\right]\sigma_h^2 + \left[\lambda^2 + h^2\tan^2\gamma_t\right]\sigma_{\alpha_t}^2$$

$$+ h^2\sec^4\gamma_t\left[\frac{h^2\tan^2\gamma_t}{l^2} + 1\right]\sigma_{\gamma_t}^2 + r^2\varepsilon^2 \tag{14}$$

where $\lambda^2 = r^2 - h^2 \sec^2\gamma_t$.

As one might expect, the variances in x and in y depend on whether the object is to port or starboard of the towfish ($k = \pm 1$), but the variance in position does not depend on k.

Ship frame of reference

The variables in the transformation to the ship frame from the towfish frame are the additional variables which enter into the calculation of x_s and y_s, namely the location of the towfish as measured by an acoustic baseline system, layback, and ship's heading (yaw). The position accuracy of the acoustic baseline is usually stated by the manufacturers as a percentage of the range between the ship and the object being tracked.

The addtitional variance due to the transformation is found by applying Eq, 12 to the transformation equation, Eq. 7. The partial derivatives of x_s and y_s with respect to B_x, B_y, and α_s are easily found. The error in ship heading, $\Delta\alpha_s$, is known. Since B_x and B_y are measured by the ship-based system, they are in a coordinate system aligned with the instantaneous ship heading. The acoustic system fixes the remote body within a position error ΔP_{bl}, which will be considered to be a circle around the mean position of the body. Thus $\Delta B_x = \Delta B_y = \Delta P_{bl}$. The quantity sought, $\sigma^2_{t\to s}$, is found to be

$$\sigma^2_{t\to s} = 2\,\sigma^2_{P_{bl}} + \left[B_x^2 + B_y^2\right]\sigma^2_{\alpha_s} \qquad (15)$$

The term in brackets is the square of the horizontal distance between the ship and the body, called the layback, L.

World frame of reference (UTM)

The variables which relate the ship frame of reference to UTM coordinates are the ship position plus the specification in the ship frame

of the vector from the acoustic baseline to the navigational antenna. Assuming that the horizontal and vertical shipboard distances between the baseline and the antenna can be measured with insignificant error, there remain four errors: the position accuracy of the surface navigational system and the errors in the ship's instantaneous heading, pitch, and roll. Since ship pitch and roll are measured in similar, if not identical, ways, the errors in pitch and roll will be assumed to be equal.

The variances in the horizontal components of the vector between the known points on the ship, x_b and y_b, are given, as before, by the sum of the products of the variances and the partial derivatives of x_b and y_b with respect to the three angles (Eq. 9). The variance associated with the conversion to the world frame is the sum of these variances plus the variance in the ship's navigation, resulting in the following lengthy expression:

$$\begin{aligned}
\sigma^2_{s\to w} = \sigma^2_{nav} + \sigma^2_{\alpha_s} \Big[& D_x^2 (\sin^2\alpha_s\cos^2\gamma_s + \sin^2\alpha_s\sin^2\beta_s \sin^2\gamma_s - \cos^2\alpha_s\cos^2\beta_s - \xi) \\
& + D_y^2 (\cos^2\alpha_s\cos^2\gamma_s + \cos^2\alpha_s\sin^2\beta_s \sin^2\gamma_s + \sin^2\alpha_s\cos^2\beta_s + \xi) \\
& + D_xD_y(\cos2\alpha \sin2\beta \sin\gamma + \sin2\alpha \cos^2\beta \sin^2\gamma - \sin2\alpha \sin^2\beta)\Big] \\
+ \sigma^2_{\beta_s} \Big[& D_x^2 (2 \cos^2\alpha \cos^2\beta \sin^2\gamma + \sin^2\beta - \xi) \\
& + D_y^2 (2 \sin^2\alpha \cos^2\beta \sin^2\gamma + \sin^2\beta + \xi) \\
& + D_z^2 (1 + \cos^2\beta - 2 \cos^2\beta \sin^2\gamma) \\
& + D_xD_y (2 \sin2\alpha \cos^2\beta \sin^2\gamma + \bar{\xi}) \\
& + D_xD_z (2 \cos\alpha \cos^2\beta \sin2\gamma - \sin\alpha \sin2\beta \cos\gamma) \\
& + D_yD_z (2 \sin\alpha \cos^2\beta \sin2\gamma + \cos\alpha \sin2\beta \cos\gamma)\Big] \qquad (16)
\end{aligned}$$

where $\xi = 1/2 \sin2\alpha \sin2\beta \sin\gamma$, $\bar{\xi} = 1/2 \cos2\alpha \sin2\beta \sin\gamma$, and $\sigma_{\beta_s} = \sigma_{\gamma_s}$

Total Variance in Position

The total variance in the UTM coordinates of the bottom object is the variance in the towfish frame, plus the variance in conversion to the ship frame, plus the variance in conversion to the world frame.

$$\sigma^2_{utm} = \sigma^2_{tf} + \sigma^2_{t\to s} + \sigma^2_{s\to w} \qquad (17)$$

Each of these terms has been presented above.

Application to a Typical Side-Scan Deployment

Variables and errors which describe two typical side-scan route survey deployments are listed in Table 1. Obviously many other sets of values could have been chosen. In particular, the horizontal beamwidth is representative of a 500 kHz system. The range and height resolution are considerably larger than the theoretical value based on the pulse length; they were chosen as indicative of the accuracy with which the relevant distances can be read from paper records. The accuracy of an ultra-short baseline system and errors in yaw, pitch, and roll were taken from manufacturer's literature and converted to standard deviations as discussed above.

To make the connection between variances and observed errors, a distribution must be assumed, and there is no reason to assume other than normal distributions for all variables, on average. For the normal distribution, one approach, the one which will be used here, is to take the error to be the extreme of the band into which 19 out of 20 observations fall, thus 1.5 × the standard deviation. For example, current operating procedure for a naval gyrocompass is that an error of $\pm 1.5^{\circ}$ is normal and acceptable; the standard deviation is therefore $\sigma = 1^{\circ}$. The other errors are similarly based on the instruments, rather than the data, and are therefore stationary.

These values are combined with the theory presented above to:

1. Indicate the amount contributed by the error in each variable to the overall standard deviation in position,
2. Demonstrate graphically the improvement or degradation in overall position standard deviation which would result from increased or decreased accuracy for each variable.

The calculations were done by means of a FORTRAN program which evaluated the expressions for variances given above for scenario data held in a data file. Provision was made for varying each error in turn by a factor of 10 above

TABLE 1

Standard Deviations (SD) in Positions of Bottom Objects in Typical Side-scan Deployments

	Typical Value or Standard Deviation (SD)		Position SD (m) if all other SDs were zero	
Scenario:	1	2	1	2
Horizontal beam width (degrees)	0.2	0.2	0.18	0.12
Sonar range, towfish to object (m)	50.0	35.0		
Range resolution (m)	0.5	0.5	0.51	0.51
Height above bottom (m)	10.0	5.0		
Height resolution (m)	0.2	0.2	0.04	0.12
Towfish yaw angle (degrees)	0.0	10.0		
SD in towfish yaw (degrees)	1.0	1.0	0.85	0.61
Towfish pitch angle (degrees)	0.0	30.0		
SD in towfish pitch (degrees)	0.5	0.5	0.09	0.06
Layback (m)	100.0	100.0		
Sonar range, ship-towfish (m)	120.0	120.0		
Accuracy of acoustic baseline (% of sonar range)	1.2	1.2	2.04	2.04
Ship roll angle (degrees)	0.0	20.0		
Ship pitch angle (degrees)	0.0	-20.0		
SD in ship roll or pitch (degrees)	0.5	0.5	0.06	0.10
Ship yaw (degrees)	0.0	-10.0		
SD in ship heading (degrees)	1.0	1.0	1.75	1.75
SD in ship position (m)	3.0	3.0	3.00	3.00
Distances between baseline and antenna:				
Fore-aft (m)	10.0	10.0		
Lateral (m)	0	0		
Vertical (m)	5.0	5.0		
Total Standard Deviation in Position (m)			4.15	4.11

and 4 below the listed value, and for plotting the results using the DREP MAGIC package. Individual contributions to the total error were found by simply setting all the other errors to zero in the data file; these results were checked by verifying that the sum of the individual variances equalled the variance calculated with all errors at their scenario values (i.e. total standard deviation equal to the sum of the squares of the individual standard deviations).

Ranking the errors in the operational variables as to their importance in the total standard deviation in position can be done from the results shown in Table 1. In these two scenarios, the ship and towfish attitudes, range, and height were varied, to illustrate the errors which would be found if the same instruments, with constant instrumental errors, were used in the two cases. The total standard deviations in position of the bottom object are almost identical for these two scenarios, namely

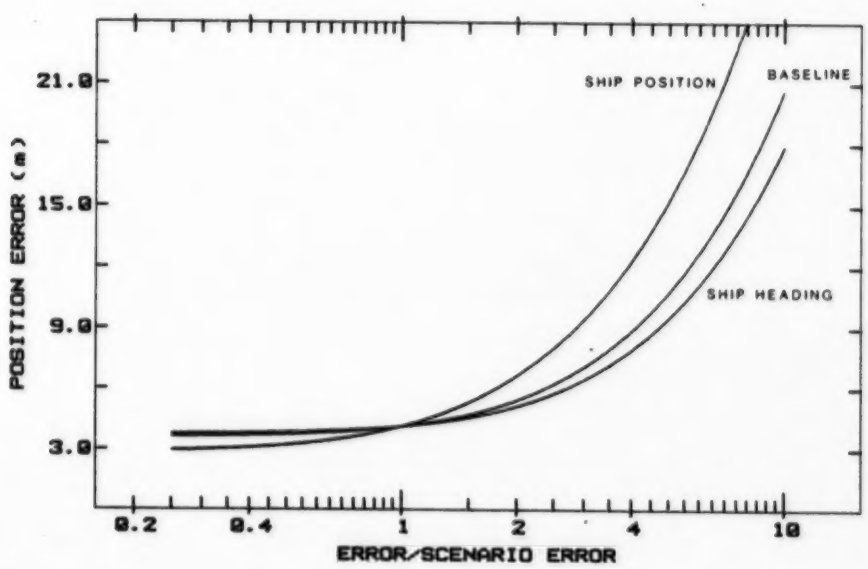

Figure 4. Total standard deviation in the position of a bottom object for changes in the accuracies of fixing the ship position, of the acoustic baseline, and of the ship's heading by the factor shown relative to the values in Table 1. Each apparent curve is actually a pair of curves, the lower curve being associated with scenario #2 of Table 1.

4.15 m for the even-keel scenario (#1) and 4.11 m for scenario #2. In both scenarios the most important errors are, in order, the error in ship position, the accuracy of the acoustic baseline, the error in the ship's heading, the error in the towfish heading, and the range resolution of the sonar records. All other errors are insignificant in comparison.

In a practical sense, the important question is not just the contribution of the error in a particular variable to the total position error, but rather the potential benefit from improvements in measurements of that variable. One might ask, for example, how useful it would be to develop an acoustic baseline system 2, 5, or 10 times more accurate than current systems. Such questions can be answered, for the typical scenarios described in Table 1, by reference to Figures 4, 5, and 6. Each of the curves in these Figures is a plot of the total position standard deviation

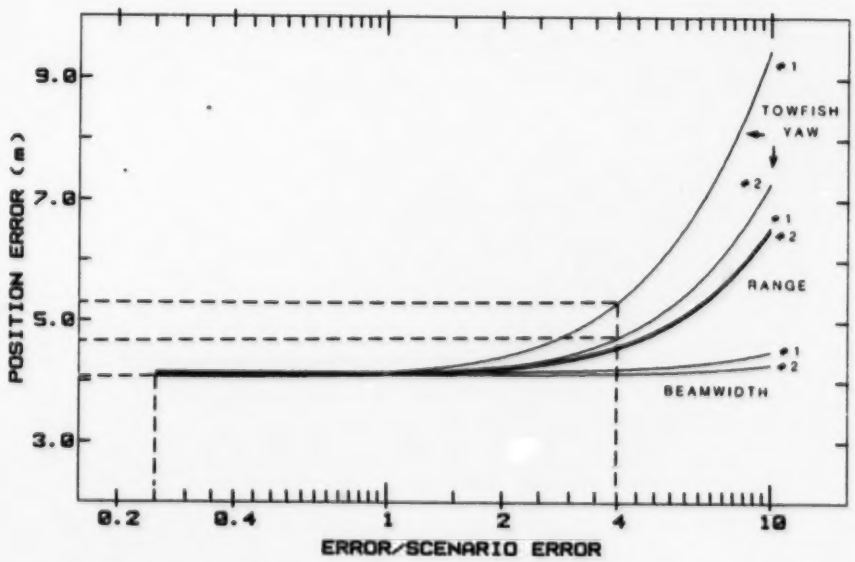

Figure 5. Total standard deviation in the position of a bottom object for changes in the accuracies of measuring towfish yaw and towfish-to-object range and for changes to the horizontal beamwidth by the factor shown relative to the values in Table 1. The scenario numbers are also shown. The dashed lines illustrate the examples discussed in the text.

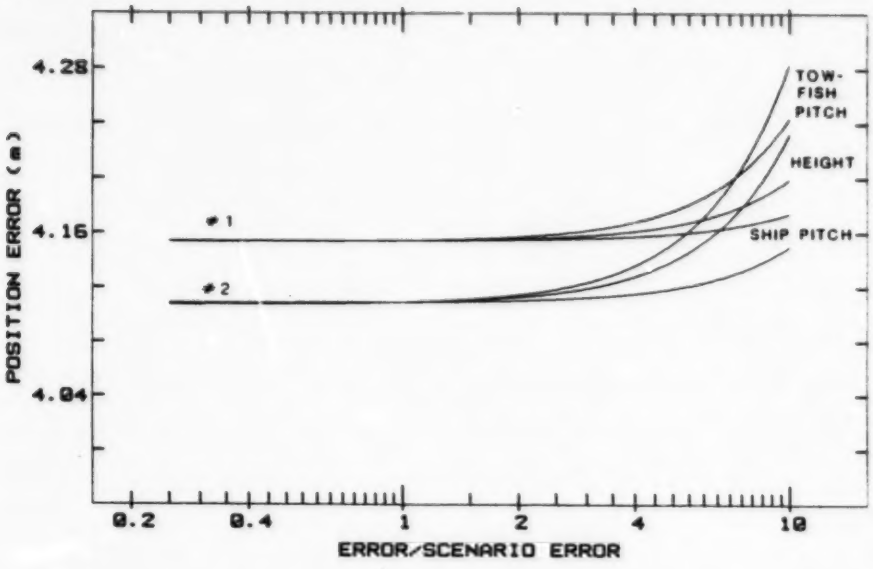

Figure 6. Total standard deviation in the position of a bottom object for changes in the accuracies of measuring towfish and ship pitch and towfish height above bottom by the factor shown relative to the values in Table 1.

against the error named, plotted as that error divided by the scenario value for that error, on a logarithmic scale. For example, degrading the accuracy with which towfish yaw is measured (Figure 5) from $\pm 1^{\circ}$ as specified in the scenarios to $\pm 4^{\circ}$ would raise the position error from 4.1 m to 4.7 or 5.3 m while improving towfish yaw accuracy by a factor of 4 (error/scenario error = 0.25) decreases the position error only from 4.1 to 4.0 m. The scenarios demonstrate different sensitivities to degradations in yaw accuracy, due primarily to the different towfish-object ranges. With these exceptions, though, the plots show little change from one scenario to the other, indicating that they would provide a good indication of the relative importance of error improvements and degradations in many scenarios.

DERIVATION OF THE ANGULAR VARIABLES OF TAP DURING MINEX 87

Introduction

MINEX 87 was a route survey/minehunting exercise conducted in the approaches to Halifax harbour in June 87. Side-scan sonar was the detection means used in this exercise, and the main aim of the exercise was to demonstrate the ability to classify mine-like objects (MLO) as mine or non-mine by comparison of the post-mine-laying images with the original (route survey) images. To this end, many passes over the area were made, towing a side-scan towfish and continually measuring the ship position, the distance to the towfish, and the depth of the towfish, as described below. The side scan system and recorder, made by Klein Inc., generated sonar images on paper, by means of a marking electrode which traversed the width of that part of the paper in the same time required for the sonar pulse to complete a return trip to the maximum range, and whose intensity was proportional to the sonar return. The distance to the first return is a measure of the towfish height above the bottom, while the rest of the record is a sonar image spaced by slant, rather than horizontal, range. What was not measured in MINEX 87, however, were the angular variables, specifically the direction from the ship to the towfish and the attitudes of both the ship and the towfish; these were either estimated or assumed to be zero. After comparison of the pre- and post-mine-laying records, the coordinates of suspected mines were tabulated and a small vessel equipped with a remotely operated underwater vehicle called a DART was dispatched to obtain video records of the mine, using a video camera on the DART and a VCR in the vessel.

Because this was the first such exercise or trial conducted in a current-free area in many years, it offered an opportunity to apply TAP theory. This section describes the method which was used to calculate the missing TAP variables (all of which were angles) from the sonar records.

Ship positions during MINEX 87 was obtained from the Precise Navigation System (PINS) which received its inputs from a Racal Microfix

microwave positioning system with four shore stations. PINS is an extremely useful and versatile system for manipulation and display of navigational data. During MINEX 87 it calculated, every 2 s, the vector sum of the coordinates of the navigational antenna on the ship's mast, the vector to the shipboard acoustic transponder, and the vector between the transponders on the ship and on the towfish. At the time of this exercise, however, we did not have available an acoustic baseline positioning system and thus could not measure the direction from the ship to the towfish, as has been assumed up to this point in this Memorandum. Instead, the direction of the underwater vector was estimated by a "follow the dog" algorithim described below. The towfish depth and the distance from the ship were measured by a Mesotech Ltd Range-Depth Unit, which measures range by round-trip time-of-flight, with a following pulse generated by the underwater module at a delay proportional to depth. (DREP has recently purchased a Trackpoint II ultrashort-baseline acoustic system which will be used in future exercises to measure range, depth, and direction.)

Towfish attitude was not measured during MINEX 87. During launch and recovery of the towfish, when it could be seen just below the surface (with the downweights deployed), it was observed that the pitch was bow upward. Because of the two-part tow, consisting of a down-weight connected by a neutrally buoyant line to the neutrally buoyant towfish, towfish attitude was expected to be little affected by wave action. This was, in fact, the case, as evidenced by the absence of along-track smearing and other effects of changing pitch and yaw on the sonar records. Thus it was reasonable to expect that towfish yaw, pitch, and roll had well defined mean values with reasonable variances. (DREP has recently constructed a towfish instrumented to measure attitude.)

Ship attitude was not measured during MINEX 87. However the previous section shows that ship pitch, roll, and yaw are the least important variables in the definition of the side-scan deployment, and ship attitude will not be considered in this section.

This section is concerned, then, with using TAP theory to derive the important angular variables involved in obtaining the bottom coordinates

which might have been, but were not, measured during MINEX 87. The sonar records and the logged navigational data will be used to do so. A legacy of MINEX 87 was a set of sonar records of the exercise area in which each bottom object appeared at least eight times. This number of images resulted from two surveys of the area (before and after mines were laid) each with N/S and E/W lanes and with 100% overlap. Some objects appeared on a run and on both its neighbours, and some preliminary runs were done before the survey proper started; thus some objects appeared more than eight times. The navigational data exists in two forms: printed logs with 20 s between sets of values and tapes with 2 s between set of values. Only the printed logs were used, with interpolation, because the increase in accuracy afforded by the tapes would not have justified the time required to find the data.

Coordinates of Bottom Objects

The sonar records from MINEX 87 consist of paper rolls from the Klein recorder. The recorder was operated in three-channel mode, namely the port and starboard images and the annotation supplied by PINS. The annotation consists of a fiducial mark, the event number, and the Easting and Northing of the towfish, calculated as described above (also see note below). The images have not been corrected for slant range. The spacing between annotations is 20 s, thus the event numbers, defined as the number of seconds since 0000 h on the first day of the exercise, show increments of 20.

Five distinctive objects which appeared in eight or more images were selected, and their coordinates were digitized using a Summagraphics MM1103 digitizing tablet and a Compaq 286 Personal Computer (PC). The software interpolated between the fiducial marks on the sonar record, derived the horizontal offset from the slant range, and calculated the coordinates of the bottom object. An interpolation between the annotated event numbers was also done. This exercise produced a list of object and towfish coordinates, towfish headings and heights, offsets (horizontal distance between towfish and object, positive to port, negative to

starboard) and event numbers giving the time when the object was abeam of the towfish. Most of these values are given in Table 2. Note that the values for offset generated by this program and listed here have been calculated assuming zero towfish pitch, i.e. as the square root of the difference between the squares of the sonar range and the towfish height.

Most of the selected objects were low lying, such as scrape intersections, so that the centre of the image would correspond to the centre of the object regardless of the position of the towfish. In digitizing the rock features, the centre of the rock instead of the centre of the sonar highlight was selected as nearly as possible.

Since the layback is required in the analysis, the ship position at each of these times was calculated by interpolation in the PINS log, which lists all the positioning variables at 20 s intervals.

Early comparisons during MINEX 87 of neighbouring anti-parallel sonar records disclosed a consistent difference in the cross-track coordinates of about 10 m. Consideration of the ship's headings showed that the towfish was tracking the ship about 5 m to port. Since the rest of the work in MINEX 87 would rely heavily on comparison of neighbouring records, the data in PINS was changed at that time from reality to a towpoint 5 m to starboard of its actual location. Since this study is an examination of the object coordinates referenced to the coordinates of the navigational antenna, this correction had to be removed. In particular, all data with an event number exceeding 800000 include the artificial offset. Removal of the artificial offset was done by a simple 5 m displacement to port. Although this seems obvious, a complete justification requires some explanation.

There are three points on the ship which enter into the calculation of the towfish position: the navigational antenna, the shipboard towfish-range transducer, and the towpoint. The first two are the ends of the vectors which are added to calculate the towfish coordinates, as described above. The towpoint is used in the "follow the dog" algorithim which is used to estimate the off-stern angle, in the absence of

TABLE 2

COORDINATES*, VARIABLES, AND DERIVED VALUES OF BOTTOM OBJECTS SEEN IN MINEX 87

Easting	Northing	Offset	Layback	Height	Towfish Heading	S	I	Event-number
Huge rock								
211.3	1682.8	-18.3	113.2	6.6	0.0	0.89	0.16	389853
212.3	1672.8	-11.9	136.9	7.0	1.3	-9.09	-1.05	652876
204.4	1687.4	-28.6	148.0	7.0	182.9	-5.13	-7.33	653488
212.8	1672.1	23.5	126.4	6.2	358.6	-9.84	-1.10	1249144
214.8	1671.5	-14.8	144.5	7.7	0.0	-10.41	-3.34	1246738
215.8	1685.4	30.6	129.8	5.8	270.0	-4.34	-3.49	1278546
215.5	1685.3	-10.3	144.5	5.8	270.0	-4.04	-3.39	1279943
208.7	1685.7	-37.7	90.2	9.2	91.7	-2.87	3.71	908784
207.8	1689.0	1.0	122.2	9.0	90.0	-3.66	7.09	906994
211.2	1687.1	41.1	122.8	8.8	90.0	-0.26	5.19	905254
Scrape intersection near circular scrape								
742.8	537.9	-29.0	155.5	10.1	268.7	-3.35	0.60	1351689
733.7	538.8	15.8	143.4	8.0	90.0	-5.74	0.37	825215
743.4	545.6	19.0	170.8	8.4	271.1	-3.82	-7.25	816936
750.7	529.7	12.2	150.6	5.5	1.4	-8.45	-11.47	738470
736.3	532.8	-26.7	130.8	8.2	180.0	5.63	-3.14	1270828
732.4	538.9	-25.1	186.6	8.9	90.0	-7.04	0.47	1359580
738.8	543.1	19.8	121.2	11.9	180.0	-4.67	-0.64	752304
737.4	540.6	15.4	122.7	10.3	180.0	-2.17	-2.04	1268269
Intersection of scrape and several parallel scrapes								
186.2	618.1	10.3	139.0	4.9	4.5	-1.91	-4.37	389332
185.6	611.9	10.5	149.8	6.0	358.7	-8.53	-3.42	1246247
182.4	613.1	17.1	157.5	6.2	358.7	-7.26	-0.25	652378
176.5	626.0	20.5	147.1	9.2	180.0	-5.65	-5.49	651155
174.0	624.5	13.0	194.5	6.9	180.0	-4.15	-7.99	388265
181.8	625.7	18.8	137.8	8.1	180.0	-5.35	-0.19	1254804
177.7	612.3	-25.4	167.1	10.7	88.7	-4.47	-7.95	1353881
187.9	630.2	12.4	138.5	6.8	272.7	-5.44	-10.12	822181
177.9	618.7	18.6	147.8	8.5	88.6	-1.55	-4.13	1355452
189.9	623.0	-23.6	157.4	6.3	269.0	-7.96	-2.51	819057

TABLE 2 continued

COORDINATES*, VARIABLES, AND DERIVED VALUES
OF BOTTOM OBJECTS SEEN IN MINEX 87

Easting	Northing	Offset	Layback	Height	Towfish Heading	S	I	Event-number
Three small rocks in line								
224.9	1175.1	-33.0	126.4	6.6	1.0	-2.05	-0.32	389615
229.9	1170.5	-33.0	137.7	7.9	1.3	-6.53	-5.44	1246509
225.5	1169.0	-24.5	145.0	5.4	0.0	-8.16	-0.89	652645
218.4	1177.1	-20.5	156.2	8.1	88.8	-6.21	0.07	839796
228.4	1183.9	9.1	151.4	8.1	270.0	-3.79	-6.74	1339090
220.9	1177.7	16.4	147.5	8.3	91.3	-3.72	0.46	838059
225.6	1187.0	-34.2	141.3	8.4	268.8	-1.19	-9.82	1340572
228.1	1170.9	7.8	176.0	6.3	358.9	-6.32	-3.37	1248912
219.8	1183.2	-14.4	145.8	9.3	178.6	-6.16	-4.66	653732
Right-angled triangle of rocks								
223.1	748.6	11.9	179.4	7.4	0.0	-6.16	-4.10	1248712
216.9	757.2	36.3	148.1	10.5	88.5	-2.04	2.49	1350826
219.5	762.3	-4.7	150.5	8.6	90.0	0.5	7.54	1349285
218.4	752.9	-24.3	159.6	6.6	1.5	-1.88	0.55	389398
213.6	757.0	-47.1	147.0	9.5	91.7	-5.46	2.08	1347829
213.2	763.6	-22.8	128.7	7.5	180.0	-8.84	-5.80	653943
222.8	748.8	-24.0	146.6	6.3	1.4	-5.87	-3.94	652444
224.5	747.7	-28.5	180.7	6.6	1.2	-6.95	-5.65	1246312

* Heading is in degrees from True North, all other values (except event-number) are in metres. Coordinates are in the local grid used on MINEX 87. Offset is the horizontal distance between the towfish and the object, has been calculated for this Table assuming zero towfish pitch, and is positive to port, negative to starboard.

measurements thereof. In this algorithim, the towfish coordinates are taken to be directly on the line between the towpoint and the previous towfish coordinates, at the measured horizontal distance (layback). The effect of an artificial offset can be removed by simple subtraction, then, only if the ship has been following a straight course for a reasonable number of iterations, perhaps 10. Since the closest object studied was 160 m from the area border, with the ship having taken up its course some seconds before starting the lane and with a typical speed of 2 m/s and an iteration period of 2 s, it can be concluded that for all the objects studied the course was sufficiently straight.

Analysis

The analysis proceeds as follows:

1. The measured coordinates differ from the actual coordinates by a combination of random errors in the measured variables plus, possibly, systematic variations due to near-constant values of the unmeasured variables.
2. The mean value of the random errors is assumed to be zero, and the sample size used here is large enough to give a mean close to zero. Ship pitch, roll, and yaw are assumed to be random with zero mean.
3. The unmeasured variables are all angles, namely the off-stern angle and towfish yaw and pitch. Each of these angles produces the opposite effect on the measured object coordinates when the direction of travel is reversed.
4. Thus the actual object coordinates are best estimated by the arithmetic mean of the measured coordinates, and the estimate will be accurate if the sample is sufficiently large. The correct coordinates would have been recorded on every pass if each of these angles were zero and there were no errors in the measured variables. Calculation of the mean value of the unmeasured angles may be possible based on an analysis of the differences between the recorded and actual coordinates as functions of the variables measured at each pass.

For the case of a towfish position specified by a layback, a depth, and an off-stern angle, the vector $\underline{B}$ is replaced, in Eqs. 7 and 10, by its polar representation to give

$$\begin{aligned} E_t &= E_s - (L\cos\delta + D_x)\sin\theta - (-L\sin\delta)\cos\theta \\ N_t &= N_s - (L\cos\delta + D_x)\cos\theta + (-L\sin\delta)\sin\theta \end{aligned} \tag{18}$$

where (E_t, N_t) are the UTM coordinates of the towfish, L is the layback (horizontal distance between the ship and the towfish), δ is the off-stern angle (Figure 7), and the other variables are as before. Combined with equation 6, this equation gives

$$\begin{aligned} E_o = E_s &- (D_x + L\cos\delta + k\lambda\sin\alpha_t - h\cos\alpha_t\tan\gamma_t)\sin\theta \\ &- (-L\sin\delta + k\lambda\cos\alpha_t + h\sin\alpha_t\tan\gamma_t)\cos\theta \\ N_o = N_s &- (D_x + L\cos\delta + k\lambda\sin\alpha_t - h\cos\alpha_t\tan\gamma_t)\cos\theta \\ &+ (-L\sin\delta + k\lambda\cos\alpha_t + h\sin\alpha_t\tan\gamma_t)\sin\theta \end{aligned} \tag{19}$$

where it is understood that the measured variables, E_s, N_s, L, λ, and h may contain random errors. As before, λ is the offset, the horizontal distance between the towfish and the object; α the ship's yaw; and θ the course. The offset, λ, is always positive and its associated index, k, is +1 if the object is to port, -1 if to starboard. Note that the values for offset

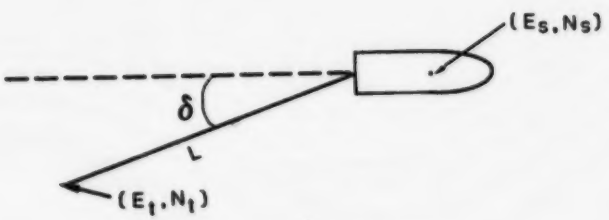

Figure 7. Use of off-stern angle and layback to specify towfish location with respect to the ship.

listed in Table 2 are for zero towfish pitch, $\gamma_t = 0$. This assumption is not correct for MINEX 87, but will be continued through most of the analysis and the necessary correction will be discussed at the end.

Based on the arguments presented above, the mean coordinates are given by the above pair of equations with no random errors and with all angles set to zero:

$$\begin{aligned} \bar{E}_o &= E_s - (D_x + L)\sin\theta - k\lambda\cos\theta \\ \bar{N}_o &= N_s - (D_x + L)\cos\theta + k\lambda\sin\theta \end{aligned} \tag{20}$$

and therefore,

$$\begin{aligned} E_o - \bar{E}_o &= S\sin\theta - T\cos\theta \\ N_o - \bar{N}_o &= S\cos\theta + T\sin\theta \end{aligned} \tag{21}$$

where S and T, which are the components of Easting and Northing excursion on the ship's track, are defined as

$$\begin{aligned} S &= L(1 - \cos\delta) - k\lambda\sin\alpha_t + h\cos\alpha_t\tan\gamma_t \\ T &= L\sin\delta + k\lambda(1 - \cos\alpha_t) - h\sin\alpha_t\tan\gamma_t \end{aligned} \tag{22}$$

The first numerical step in the analysis was to calculate the excursion of each pair of recorded coordinates from the mean coordinates of that object, and thus the S,T pair for each record. These are tabulated in Table 2.

S and T depend on six independent variables: α_t, δ, γ_t, L, h, and λ. This analysis depends on the first five of these having limited variations about a mean value. If this is so, it will be possible to calculate those mean values for the angles; if not so, no relationship will be discernible in the plots. The measured variables are known to vary in specific ranges, namely layback in the range 90 m < L < 195 m and height in the range 4.8 m < h < 12 m for all the records being examined here (i.e.

over a fraction, albeit a significant fraction, of their magnitude). It is, of course, possible to reduce these variations by selecting records with L or h in specific tighter ranges. By contrast the offset with its index, $k\lambda$, varies from -47 m to 41 m, and is therefore the "most independent" variable. Plots were therefore done of S and T against $k\lambda$, for all records (Figures 8 and 9) and for selected bands of L and h (Figures 10 and 11 are two examples).

Considering S vs. $k\lambda$, the least-squares best-fit slope with all records was 0.00 ± 0.02, where the stated uncertainty is the standard error of the slope. Plots with three selected bands of h and of L gave no case in which zero was beyond two standard deviations of the slope. Turning to T vs. $k\lambda$, the all-records slope was 0.01 ± 0.03, with the same conclusion for the plots with selected bands. Since theory gives the slopes as $\sin\alpha_t$

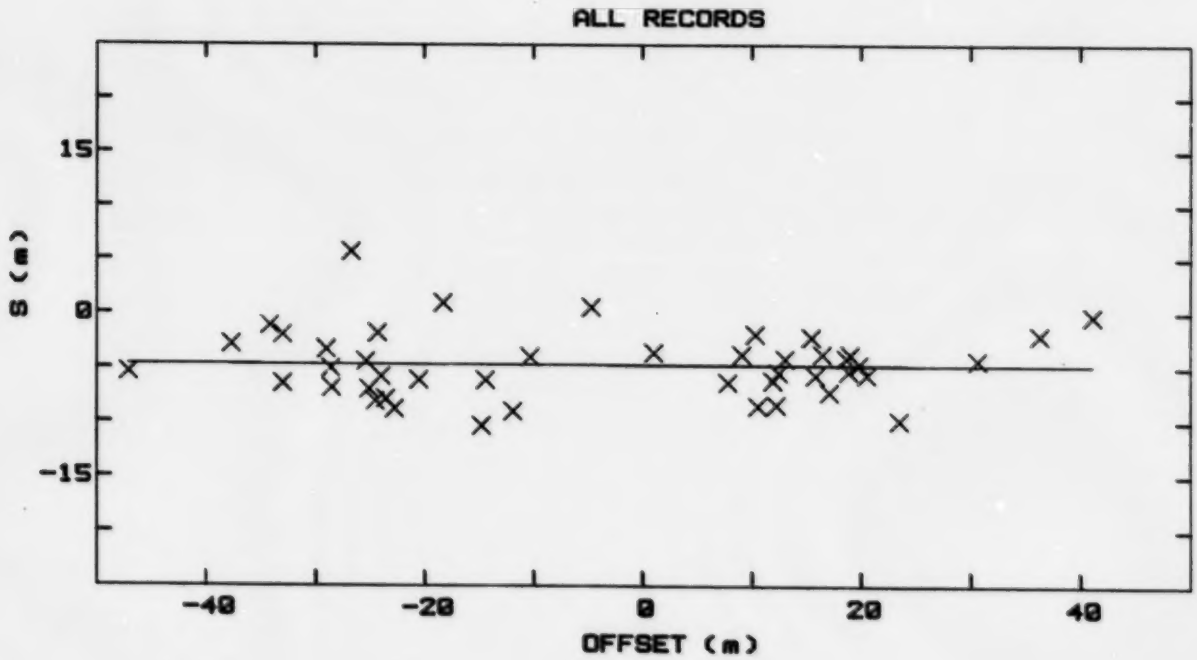

Figure 8. Values of S, calculated from Eq. 22, against offset for all 45 coordinate pairs of the five bottom features studied. The least-squares best-fit straight line is also shown. The axes are to the same scale.

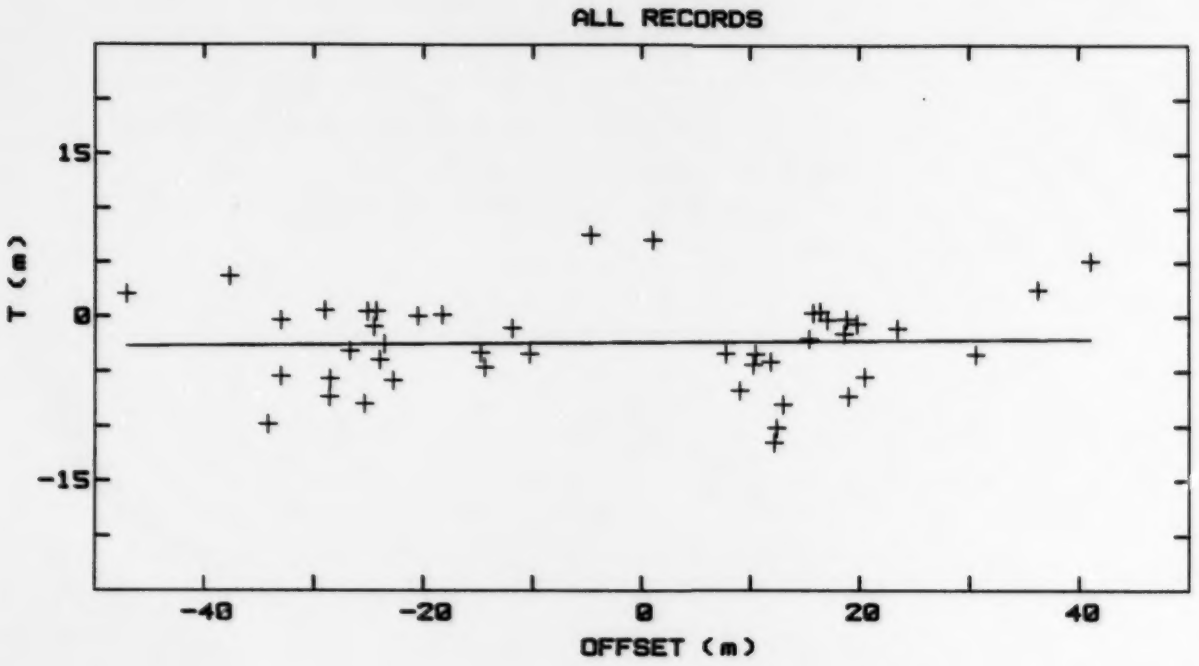

Figure 9. Values of T, calculated from Eq. 22, against offset for all 45 coordinate pairs of the five bottom features studied. The least-squares best-fit straight line is also shown. The axes are to the same scale.

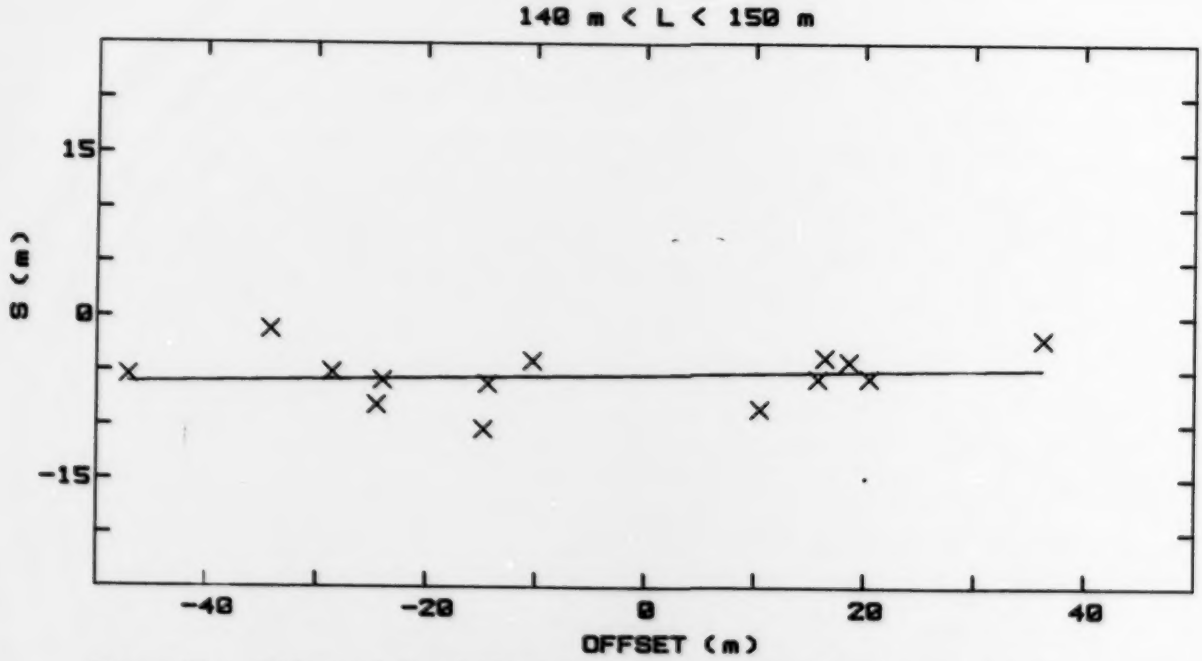

Figure 10. Values of S, calculated from Eq. 22, against offset for all the points of the original 45 for which the layback was between 140 and 150 m. The least-squares best-fit straight line is also shown.

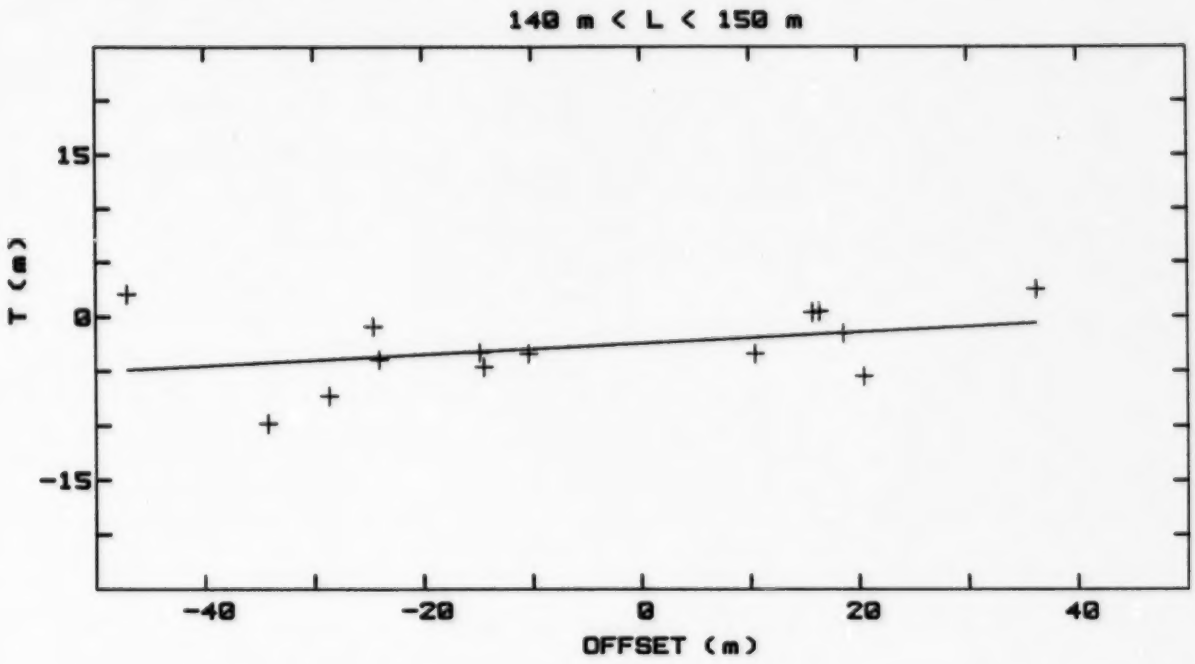

Figure 11. Values of T, calculated from Eq. 22, against offset for all the points of the original 45 for which the layback was between 140 and 150 m. The least-squares best-fit straight line is also shown.

and $(1 - \cos\alpha_t)$ for S and T respectively vs. $k\lambda$, the conclusion is that there was no mean towfish yaw, $\alpha_t = 0$. Specifically, the S vs. λ plots provide the tighter error constraint and give $\alpha_t = 0.0^o \pm 1.1^o$.

The conclusion that $\alpha_t = 0$ reduces Equations 22 to

$$\begin{aligned} S &= L\,(1- \cos\delta) + h \tan \gamma_t \\ T &= L \sin\delta \end{aligned} \qquad . \tag{23}$$

Thus the next angle that can be determined is the off-stern angle, δ, from a plot of T vs. layback, L. All records can be used in this plot, since no other variables enter the relationship, and the appropriate least-squares best-fit line is one constrained to pass through the origin. The slope of this plot (Figure 12) is -0.017 ± 0.004, giving $\delta = 1.0^o \pm 0.2^o$.

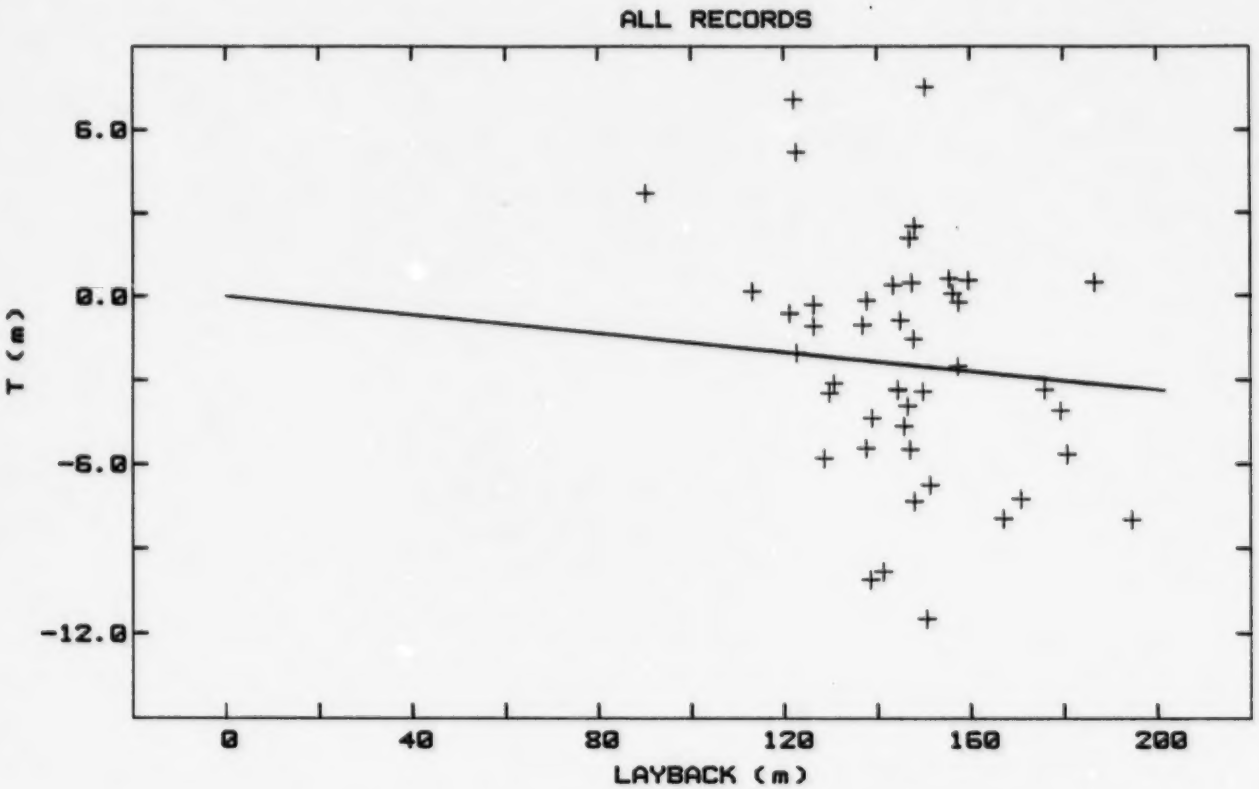

Figure 12. Values of T, calculated from Eq. 22, against layback for all 45 coordinate pairs of the five bottom features studied. The line shown is that line which passes through the origin and is the least-squares best fit.

The calculation of the final angle, towfish pitch, γ_t, is facilitated by an examination of the magnitudes of the terms in the expression for S (Eq. 23). For maximum L and minimum h, $S = 194.5[1-\cos(1^o)] + 4.9\tan\gamma_t = 0.030 + 4.9\tan\gamma_t$. For $\gamma_t \simeq 30^o$, the second term is 100 times larger than the first, and thus a plot of S vs h will, in this case, have a slope of $\tan\gamma_t$. Again the plot can be of all records and the best-fit line is forced through the origin. The resulting slope (Figure 13) is -0.57 ± 0.07, giving $\gamma_t = -30^o \pm 3^o$, and justifying the neglect of the first term.

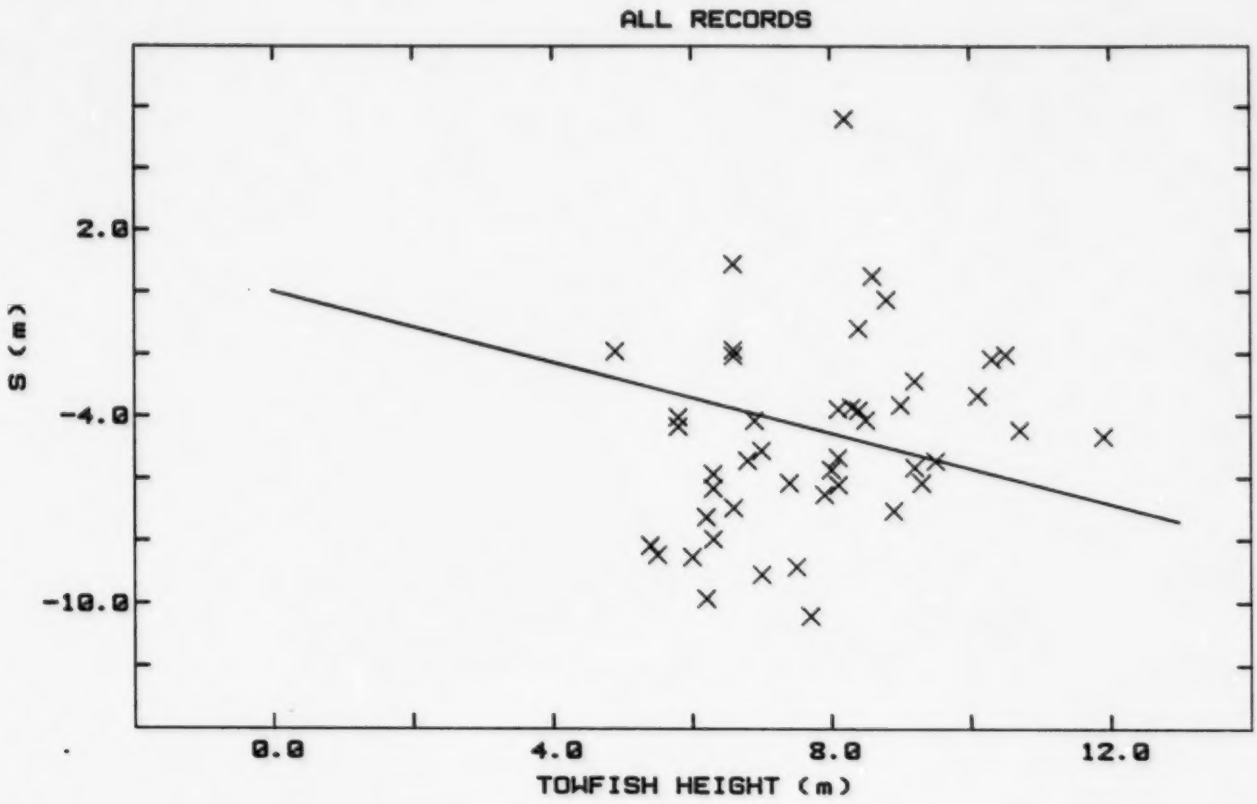

Figure 13. Values of S, calculated from Eq. 22, against towfish height for all 45 coordinate pairs of the five bottom features studied. The line shown is that line which passes through the origin and is the least-squares best fit.

The only remaining aspect of the calculation of these angles is that the offset, λ, as tabulated in Table 2 was calculated assuming zero towfish pitch, that is $\gamma_t \simeq 0$ in the expression $\lambda^2 = r^2 - h^2\sec^2\gamma_t$. In fact, γ_t has been found to be the only angle significantly different from zero. Thus it is necessary to replot S and T vs $k\lambda$, this time with offset values, λ, based on $\gamma_t = -30^o$. Implementing this change produces no change in the slopes or their standard deviations as quoted above, while the plot (Figure 14, compare with Figure 8) shows only the expected stretching along the abscissa. Thus the conclusions reached above need not be changed.

This analysis has shown that it is possible to calculate the unknown angles in towfish attitude and position based on the variations in recorded coordinates of bottom objects from the mean values of those coordinates. The plots exhibit considerable scatter due to random errors in the measured variables and to currents, but when sufficient records are treated statistically the trends predicted by TAP theory emerge clearly. During MINEX 87, the mean values of the angular variables were:

Towfish yaw	$0.0^{o} \pm 1.0^{o}$	
Towfish pitch	$30^{o} \pm 3^{o}$	bow-upward
Off-stern angle	$1.0^{o} \pm 0.2^{o}$	to port.

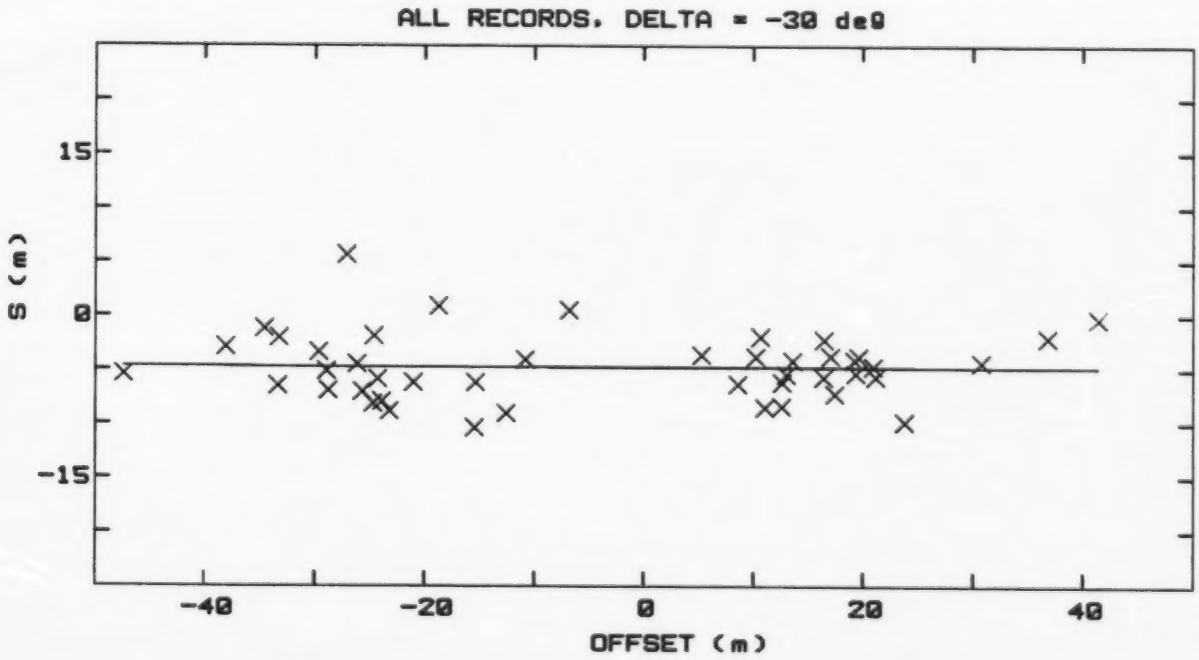

Figure 14. Values of S, calculated from Eq. 22, against offset calculated with $\gamma_t = -30^{o}$. Comparison with Figure 8 reveals no change in slope.

Comparison of Derived Angles with Observation

It is clearly important to compare the values derived above with observation and/or a physical understanding in order to put TAP theory, as presented here, on a firm footing. In making this comparison, though, one must remember that the values derived above are mean values, and thus do not relate to the action of waves on the towing vessel or to currents, since these effects averaged to zero. If tidal currents had been significant it would have been necessary to make a great many determinations of the coordinates of each object before it could be assumed that current effects would average to zero. However it was known that tidal currents in the exercise area were only a small fraction of a knot. In particular, no current drag was ever noticed on the DART, and its video showed only very gentle motion of planktonic organisms. Thus it was concluded that neither currents nor swell affected the mean values as calculated above, although they may well be responsible for much of the scatter in Figures 8-14.

Towfish yaw A more-or-less symmetrical towfish towed by the nose can take up only very small values of long-term yaw, particularly if its dimensions are large when viewed from the side. Any torque which might cause a yaw, resulting from some asymmetry, would be countered by two torques: the cable tension and longitudinal drag act on well-separated points, and the transverse drag would act well aft from the nose. Both of these restoring torques would be substantial for even very small yaw angles.

Towfish pitch, γ_t The towfish design could well have allowed the towfish to fly stably with considerable pitch. When the towfish could be seen underwater, just after launch or just before recovery, bow-up pitch was observed. The pitch angle was estimated at about 10^{o}, and could hardly have

been less to be noticeable under those circumstances. With more cable out it could have been more. In particular, the downweight, a lead sphere, had clearly contacted the bottom on numerous occasions. While much of the scraping could have happened during turns, it is quite possible that the weight was well below the towfish, quite close to the bottom, with the towfish at an height of 5-10 m. Since the towfish was only slightly positively buoyant, it would have had to have a significant angle of attack to fly stably so far above the weight.

Off-stern angle, δ The off-stern angle as calculated here is in reasonable agreement with the offset estimated from some early MINEX 87 sonar records, which showed that the towfish was tracking the ship about 5 m to port. The best-fit off-stern angle calculated here, $-1^{o} \pm 0.2^{o}$, gives, with 140 m mean layback, an offset of 2.4 ± 0.5 m to port). The cause of this angle could well have been some asymmetry in the towfish, likely in the brackets between the transducer housing and the floats. As discussed above, this would not have caused any significant towfish yaw. In the equilibrium position, the hydrodynamic transverse force caused by the asymmetry would be balanced by a small transverse component of cable tension. The resulting torque would cause only very little yaw because the restoring torques would increase very rapidly with yaw angle.

Based on observation and on rough physical models, it is seen that the angles calculated here are quite reasonable for the towfish and procedures of MINEX 87.

CONCLUSIONS

A central issue in minehunting is the accuracy with which the coordinates of a mine-like object on the sea bottom can be determined. In a typical side-scan sonar minehunting deployment, many distances and angles are involved in calculating sea bottom locations. By considering the survey problem in three parallel coordinate systems, expressions were derived which relate the coordinates of a bottom object to the variables which fully describe a side-scan sonar operation.

Error analysis of these expressions showed which variables must be measured accurately to optimize the accuracy of the derived coordinates. For typical side-scan operations, the accuracy in ship fixation, the accuracy of the acoustic baseline, the error in the ship's gyroscope, the error in the towfish heading, and the range resolution of the sonar records were found to be more important, in that order, than the effect of horizontal beamwidth and the errors in towfish pitch and ship pitch and roll. However none of these variables usually contribute the dominant error, and therefore, because of the number of variables involved, improvements in the accuracy with which any particular variable was measured gave only modest improvements in the overall accuracy.

Applying the expressions derived here to coordinates measured from sonar images from MINEX 87, off Halifax, gave values for towfish pitch, towfish yaw, and off-stern angle which were in reasonable agreement with on-site estimates. This showed that it is possible to calculate the mean values of unmeasured variables of towfish position and attitude (TAP) based on the variations in the observed coordinates of bottom features, given a sufficient number of scans at a variety of headings. The plots exhibit considerable scatter, but the trends predicted by TAP theory emerge clearly.

TABLE 3

GLOSSARY OF VARIABLES

E_o, N_o	Easting and Northing of the insonified object.
E_s, N_s	Easting and Northing of the navigational antenna on the ship.
E_t, N_t	Easting and Northing of the towfish.
D_x, D_y, D_z	Fore-aft, lateral, and vertical distances between the navigational antenna and the acoustic baseline. Signs are positive if the baseline is aft, to port, and below the antenna.
B_x, B_y, B_z	Vector to the towfish from the ship as measured by the acoustic baseline system.
x_t, y_t, z_t	Coordinates of the object in the towfish system*.
x_s, y_s, z_s	Coordinates of the object in the coordinate system* centered on the ship's acoustic baseline.
V_x, V_y, V_z	Coordinates of the acoustic baseline, in the coordinate system* centered on the ship's navigational antenna.

Note. The three coordinate systems marked with an asterisk above are all identically oriented, with the x-y plane horizontal, the x axis along the ship's course, and the z axis vertically down.

d	Depth of towfish
h	Height of towfish above bottom.
k	Index for the offset. +1 for object to port of the towfish, -1 if to starboard.
λ	Offset, horizontal distance between towfish and object.
L	Layback, horizontal distance between towfish and ship.
r	Sonar range.
R	Range between acoustic baseline on ship and towfish.
S,T	Variables related to position errors, defined by Eq. 22.

TABLE 3 continued

GLOSSARY OF VARIABLES

α	Yaw angle.
β	Roll angle.
γ	Pitch angle.

Note. These angles are those of the towfish if the subscript is t and of the ship if the subscript is s. The positive direction for each angle is according to the Cartesian right-hand rule, namely yaw to port, pitch bow-downward, and roll port-upward.

δ	Angle between bearing to towfish and dead astern.
ε	Horizontal beamwidth of the sonar.
ϕ	Vertical angle (depression) to the towfish from the ship.
θ	Ship's course.
Δv, σ_v	Error and standard deviation, respectively, in any variable v.
$\sigma_{p_{bl}}$	Standard deviation in the positioning of the towfish by the acoustic baseline.
σ_{nav}	Standard deviation in ship positioning by the surface navigation system.
σ_{tf}	Standard deviation in the position of the object in the towfish frame.
$\sigma_{t\rightarrow s}$	Additional standard deviation in the position of the object arising from transformation from the towfish to the ship frame.
$\sigma_{s\rightarrow w}$	Additional standard deviation in the position of the object arising from transformation from the ship to the world frame.

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A central issue in minehunting is the accuracy with which the coordinates of a mine-like object can be determined. This survey process is done in stages, first locating the ship and them locating the object with respect to the sonar. If side-scan sonar is used, there is the additional stage of determining the vector between the ship and the towfish. The equations which determine the chart coordinates of a bottom object from the 14 variables which describe the side-scan sonar deployment which imaged that object are derived and presented. The improvements in the accuracy of the variable is measured are then predicted. Coordinate data acquired during MINEX 87 is used to calculate the unmeasured angular variables which characterized that deployment.

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